Backgrounds of squeezed relic photons and their spatial correlations

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Abstract

We discuss the production of multi-photons squeezed states induced by the time variation of the (Abelian) gauge coupling constant in a string cosmological context. Within a fully quantum mechanical approach we solve the time evolution of the mean number of produced photons in terms of the squeezing parameters and in terms of the gauge coupling. We compute the first (amplitude interference) and second order (intensity interference) correlation functions of the magnetic part of the photon background. The photons produced thanks to the variation of the dilaton coupling are strongly bunched for the realistic case where the growth of the dilaton coupling is required to explain the presence of large scale magnetic fields and, possibly of a Faraday rotation of the Cosmic Microwave Background.

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The squeezed states formalism has been successfully applied to the analysis of tensor, scalar [1] and rotational [2] fluctuations of the metric by Grishchuk and collaborators. In this paper we want to address the possible application of the squeezed states formalism to the case of relic photons which, to the best of our knowledge, has not received particular attention. In the case of relic gravitons and relic phonons the analogy with quantum optics is certainly very inspiring. In the case of relic photons the analogy is even closer since the time variation of the dilaton coupling plays directly the rôle of the laser "pump" which is employed in order to produce experimentally observable squeezed states [3].

In a general relativistic context the evolution equations of the (Abelian) gauge field strength is invariant under conformal (Weyl) rescaling of the metric tensor. If we are in a conformally flat background of Friedmann-Robertson-Walker (FRW) type (written in conformal time η)

$$ds^2 = G_{\mu\nu} dx^{\mu} dx^{\nu} = a^2(\eta)[d\eta^2 - d\vec{x}^2], \tag{1}$$

the evolution of the Abelian gauge field strength implies that, in vacuum, the determinant of the FRW metric can be always reshuffled by appropriately rescaling the gauge fields so that the (rescaled) magnetic and electric field amplitudes will always obey, in the curved background of Eq. (1) the same Maxwell's equations they would obey in flat space-time. Consequently, in general relativity Abelian gauge fields cannot be amplified from their vacuum fluctuations [4, 5]. On the contrary, tensor fluctuations of the metric are amplified in an isotropic FRW space since their evolution equation is not invariant under Weyl rescaling of the background metric [4].

A peculiar feature of cosmological models inspired by the low energy string effective action is that the gauge coupling is not really a constant but it evolves in time [6]. Therefore, in string cosmology, photons [7] (as well as gravitons [8]) can be produced thanks to the time evolution of the gauge coupling from the electromagnetic vacuum fluctuations. A similar amplification effect exists for the case of non-Abelian gauge fields which are, however, screened at high temperature. The large (but finite) value of the conductivity in the early Universe implies that the Abelian magnetic component survives only for momenta which are much shorter than the magnetic diffusivity scale at each epoch [10, 11]. Plasma effects connected with the evolution of large scale magnetic fields have been explored with particular attention to the electroweak scale [9]. The effective action of a generic Abelian gauge field in four space-time dimensions reads

$$S = -\frac{1}{4}\int d^4x \sqrt{-G}\frac{1}{g^2} F_{\alpha\beta}\, F^{\alpha\beta}, \tag{2}$$

where $F_{\alpha\beta} = \nabla_{[\alpha} A_{\beta]}$ is the Maxwell field strength and ∇_{α} is the covariant derivative with respect to the string frame metric $G_{\mu\nu}$. In Eq. (2) $g = \exp{(\phi/2)}$ is the (four dimensional) dilaton coupling.

In the absence of a classical gauge field background, the perturbed effective Lagrangian density

$$\mathcal{L}(\vec{x},\eta) = \frac{1}{2}\sum_{\alpha}\Big[\mathcal{A}'^{2}_{\alpha} + 2\frac{g'}{g}\mathcal{A}'_{\alpha}\mathcal{A}_{\alpha}$$
$$+\Big(\frac{g'}{g}\Big)^{2}\mathcal{A}^{2}_{\alpha} - (\partial_i\mathcal{A}_{\alpha})^{2}\Big], \quad L(\eta) = \int d^3x\mathcal{L}(\vec{x},\eta), \tag{3}$$

describes the evolution of the two ($\alpha = \otimes, \oplus$) transverse degrees of freedom defined by the Coulomb gauge condition $A_0 = 0$ and $\vec{\nabla}\cdot\vec{A} = 0$ (the prime denotes differentiation with respect to conformal time). The fields $A_{\alpha} = g\mathcal{A}_{\alpha}$ have kinetic terms with canonical normalization and the time evolution

$$\mathcal{A}''_{\alpha} - \nabla^2\mathcal{A}_{\alpha} - g(g^{-1})''\mathcal{A}_{\alpha} = 0 \tag{4}$$

can be derived from the Euler-Lagrange equations By functionally deriving the the action we get the canonically conjugated momenta $\pi_{\alpha} = \mathcal{A}'_{\alpha} + (g'/g)\mathcal{A}_{\alpha}$ leading to the Hamiltonian density and to the associated Hamiltonian

$$\mathcal{H}(\vec{x},\eta) = \frac{1}{2}\sum_{\alpha}\Big[\pi^2_{\alpha} + (\partial_i\mathcal{A}_{\alpha})^2 - 2\frac{g'}{g}\mathcal{A}_{\alpha}\pi_{\alpha}\Big],$$
$$H(\eta) = \int d^3x\mathcal{H}(\vec{x}). \tag{5}$$

The operators corresponding to the classical polarizations appearing in the Hamiltonian density

$$\hat{\mathcal{A}}_{\alpha}(\vec{x},\eta) = \int d^3k\frac{1}{(2\pi)^{3/2}}\hat{\mathcal{A}}_{\alpha}(k,\eta)e^{i\vec{k}\cdot\vec{x}},$$
$$\hat{\mathcal{A}}_{\alpha}(k,\eta) = \frac{1}{\sqrt{2k}}[\hat{a}_{k,\alpha}(\eta) + \hat{a}^{\dagger}_{-k,\alpha}(\eta)],$$
$$\hat{\pi}_{\alpha}(\vec{x},\eta) = \int d^3k\frac{1}{(2\pi)^{3/2}}\hat{\pi}_{\alpha}(k,\eta)e^{i\vec{k}\cdot\vec{x}},$$
$$\hat{\pi}_{\alpha}(k,\eta) = i\sqrt{\frac{k}{2}}[\hat{a}_{k,\alpha}(\eta) - \hat{a}^{\dagger}_{-k,\alpha}(\eta)], \tag{6}$$

obey canonical commutation relations and the associated creation and annihilation operators satisfy $[\hat{a}_{k,\alpha}, \hat{a}^{\dagger}_{p,\beta}] = \delta_{\alpha\beta}\delta^{(3)}(\vec{k}-\vec{p})$. The Hamiltonian can then be written as:

$$H(\eta) = \int d^3k\sum_{\alpha}\Big[k\ (\hat{a}^{\dagger}_{k,\alpha}\hat{a}_{k,\alpha} + \hat{a}^{\dagger}_{-k,\alpha}\hat{a}_{-k,\alpha} + 1)$$
$$+\epsilon(g)\hat{a}_{-k,\alpha}\hat{a}_{k,\alpha} + \epsilon^*(g)\hat{a}^{\dagger}_{k,\alpha}\hat{a}^{\dagger}_{-k,\alpha}\Big], \quad \epsilon(g) = i\frac{g'}{g}. \tag{7}$$

The (two-modes) Hamiltonian contains a free part and the effect of the variation of the coupling constant is encoded in the (Hermitian) interaction term which is quadratic in the creation and annihilation operators whose evolution equations, read, in the Heisenberg picture

$$\frac{d\hat{a}_{k,\alpha}}{d\eta} = -ik\hat{a}_{k,\alpha} - \frac{g'}{g}\hat{a}^{\dagger}_{-k,\alpha},$$
$$\frac{d\hat{a}^{\dagger}_{k,\alpha}}{d\eta} = ik\hat{a}^{\dagger}_{k,\alpha} - \frac{g'}{g}\hat{a}_{-k,\alpha}. \tag{8}$$

The general solution of the previous system of equations can be written in terms of a Bogoliubov-Valatin transformation

$$\hat{a}_{k,\alpha}(\eta) = \mu_{k,\alpha}(\eta)\hat{b}_{k,\alpha} + \nu_{k,\alpha}(\eta)\hat{b}^{\dagger}_{-k,\alpha}$$
$$\hat{a}^{\dagger}_{k,\alpha}(\eta) = \mu^{*}_{k,\alpha}(\eta)\hat{b}^{\dagger}_{k,\alpha} + \nu^{*}_{k,\alpha}(\eta)\hat{b}_{-k,\alpha} \tag{9}$$

where $\hat{a}_{k,\alpha}(0) = \hat{b}_{k,\alpha}$ and $\hat{a}_{-k,\alpha}(0) = \hat{b}_{-k,\alpha}$. Unitarity requires that the two complex functions $\mu_k(\eta)$ and $\nu_k(\eta)$ are subjected to the condition $|\mu_k(\eta)|^2 - |\nu_k(\eta)|^2 = 1$ which also implies that $\mu_k(\eta)$ and $\nu_k(\eta)$ can be parameterized in terms of one real amplitude and two real phases

$$\mu_k = e^{i\theta_k}\cosh r_k, \qquad \nu_k = e^{i(2\phi_k - \theta_k)}\sinh r_k, \tag{10}$$

(r is sometimes called squeezing parameter and ϕ_k is the squeezing phase; from now on we will drop the subscript labeling each polarization if not strictly necessary). The total number of produced photons

$$\langle 0_{-k}0_k|\hat{a}^{\dagger}_k(\eta)\hat{a}_k(\eta) + \hat{a}^{\dagger}_{-k}\hat{a}_{-k}|0_k 0_{-k}\rangle = 2\ \overline{n}_k. \tag{11}$$

is expressed in terms of $\overline{n}_k = \sinh^2 r_k$, i.e. the mean number of produced photon pairs in the mode k. Inserting Eqs. (9),(10) and (11) into Eqs. (8) we can derive a closed system involving only the $\overline{n}_k$ and the related phases:

$$\frac{d\overline{n}_k}{d\eta} = -2f(\overline{n}_k)\frac{g'}{g}\cos 2\phi_k, \tag{12}$$
$$\frac{d\theta_k}{d\eta} = -k + \frac{g'}{g}\frac{\overline{n}_k}{f(\overline{n}_k)}\sin 2\phi_k, \tag{13}$$
$$\frac{d\phi_k}{d\eta} = -k + \frac{g'}{g}\frac{df(\overline{n}_k)}{d\overline{n}_k}\sin 2\phi_k, \tag{14}$$

where $f(\overline{n}_k) = \sqrt{\overline{n}_k(\overline{n}_k + 1)}$.

In quantum optics [12] the coherence properties of light fields have been a subject of intensive investigations for nearly half a century. Magnetic fields over galactic scales have typical frequency of the order 10^{-14}–10^{-15} Hz which clearly fall well outside the optical range. Thus, the analogy with quantum optics is only technical. The same quantum optical analogy has been successfully exploited in particle [13] and heavy-ions physics [14] of pion correlations (in order to measure the size of the strongly interacting region) and in the phenomenological analysis of hadronic multiplicity distributions.

The interference between the amplitudes of the magnetic fields (Young interferometry [15], in a quantum optical language) estimates the first order coherence of the magnetic background at different spatial locations making use of the two-point correlation function whose trace over the physical polarizations and for coincidental spatial locations is related to the magnetic energy density. Recall that in our gauge

$$\hat{B}_k(\vec{x},\eta) = \frac{ig}{(2\pi)^{3/2}a^2(\eta)}\sum_\alpha e_i^\alpha \int \frac{d^3k}{\sqrt{2k}} k_j \epsilon_{jik}[\hat{a}_{k,\alpha}e^{i\vec{k}\cdot\vec{x}} - \hat{a}^\dagger_{k,\alpha}e^{-i\vec{k}\cdot\vec{x}}]. \tag{15}$$

By summing up over the polarizations according to

$$\mathcal{K}_{ij} = \sum_\alpha e_i^\alpha(k) e_j^\alpha(k) = \Big(\delta_{ij} - \frac{k_i k_j}{k^2}\Big), \tag{16}$$

we get that

$$\mathcal{G}_{ij}(\vec{r},\eta) = \langle 0_{-k}0_k|\hat{B}_i(\vec{x},\eta)\hat{B}_j(\vec{x}+\vec{r},\eta)|0_k 0_{-k}\rangle \tag{17}$$

can be expressed, using Eqs. (9) and (10)

$$\mathcal{G}_{ij}(\vec{r}) = \int d^3k \mathcal{G}_{ij}(k) e^{i\vec{k}\cdot\vec{r}}. \tag{18}$$

where

$$\mathcal{G}_{ij}(k,\eta) = \frac{g^2(\eta)\mathcal{K}_{ij}}{2(2\pi)^3 a^4(\eta)} k[2\sinh^2 r_k + \sinh 2r_k \cos 2\phi_k] \tag{19}$$

(the vacuum contribution, occurring for $r_k = 0$, has been consistently subtracted). The intercept for $\vec{r} = 0$ of the two-point function traced with respect to the two polarizations is related to the magnetic energy density

$$\frac{d\rho_B}{d\ln\omega} \simeq \frac{g^2(\eta)\omega^4}{2\pi^2}[2\sinh^2 r_k + \sinh 2r_k \cos 2\phi_k] \tag{20}$$

(where $\omega = k/a$ is the physical frequency). The two-point function and its trace only depend upon $\overline{n}_k$ and upon ϕ_k. Since Eqs. (12) and (14) do not contain any dependence upon θ_k we can attempt to solve the time evolution by solving them simultaneously. In terms of the new variable $x = k\eta$ Eqs. (12) and (14) can be written as

$$\frac{d\phi_k}{dx} = -1 + \frac{d\ln g}{dx}\frac{df(\overline{n}_k)}{d\overline{n}_k}\sin 2\phi_k, \tag{21}$$

$$\frac{d\overline{n}_k}{d\ln g} = -2f(\overline{n}_k)\cos 2\phi_k, \tag{22}$$

If $|(d\ln g/dx)(df(\overline{n}_k)/d\overline{n}_k)\sin 2\phi_k| > 1$, then Eqs. (21) and (22) can be written as

$$\frac{du_k}{d\ln g} = 2\frac{df(\overline{n}_k)}{d\overline{n}_k}u_k, \quad \frac{d\overline{n}_k}{d\ln g} = -2f(\overline{n}_k)\frac{1-u_k^2}{1+u_k^2} \tag{23}$$

where $\phi_k = \arctan u_k$. By trivial algebra we can get a differential relation between u_k and $\overline{n}_k$ which can be exactly integrated with the result that $u_k^2 - f(\overline{n}_k)u_k + 1 = 0$. By inverting this last relation we obtain two different solutions with equivalent physical properties, namely

$$u_k(\overline{n}_k) = [\frac{1}{2}(\sqrt{\overline{n}_k(\overline{n}_k+1)} \pm \sqrt{\overline{n}_k(\overline{n}_k+1)-4})]. \tag{24}$$

If we choose the minus sign in Eq. (24) we obtain that $\phi_k \sim (m+1)\pi/2$, $m = 0, 1, 2...$ with corrections of order $1/\overline{n}_k$. In the opposite case $\phi_k \sim \arctan(\overline{n}_k/2)$ within the same accuracy of the previous case(i.e. $1/\overline{n}_k$). By using the relation between u_k and $\overline{n}_k$ the condition $|(d\ln g/dx)(df(\overline{n}_k)/d\overline{n}_k)\sin 2\phi_k| > 1$ is equivalent to $x\lesssim 1$, if, as we are assuming, $|g'/g|$ vanishes as η^{-2} for $\eta \to \pm\infty$ and it is, piece-wise, continuous. By inserting Eq. (24) into Eq. (21) a consistent solution can be obtained, in this case, if we integrate the system between η_f and η_i defined as the conformal times where $|(d\ln g/dx)(df(\overline{n}_k)/d\overline{n}_k)\sin 2\phi_k| = 1$:

$$\begin{aligned} &\overline{n}_k(\eta_f) \sim \frac{1}{4}\Big(\frac{g(\eta_f)}{g(\eta_i)} - \frac{g(\eta_i)}{g(\eta_f)}\Big)^2, \\ &\phi_k(\eta) \sim (m+1)\frac{\pi}{2} + \mathcal{O}(\frac{1}{\overline{n}_k(\eta)}), \quad m = 0, 1, 2... \end{aligned} \tag{25}$$

If $|(d\ln g/dx)(df(\overline{n}_k)/d\overline{n}_k)\sin 2\phi_k| < 1$ (i. e. $x > 1$) the consistent solution of our system is given by

$$\begin{aligned} &\overline{n}_k(\eta_f) = \sinh^2\left(2\int^{k\eta}\ln g(x')\sin 2x' dx'\right) \\ &\phi_k \sim -k\eta + \varphi_k, \quad \varphi_k \simeq \text{constant}. \end{aligned} \tag{26}$$

If the coupling constant evolves continuously between $-\infty$ and $+\infty$ with a (global) maximum located at some time η_r then, for $x > 1$, $\overline{n}_k \sim$ const.. Indeed by taking trial functions with bell-like shape for $|g'/g|$ we can show that $\overline{n}_k$ oscillates around zero for large ϕ_k.

Up to now our considerations were general. Let us give some specific example of our technique. In the low energy phase ($\eta < \eta_s$) of the pre-big-bang evolution the dilaton coupling is determined by the variation of the low-energy effective action [6]:

$$a(\eta) \simeq |\eta|^{-\frac{1}{\sqrt{3}+1}}, \quad g(\eta) \simeq |\eta|^{-\sqrt{3}/2} \quad \eta < \eta_s. \tag{27}$$

During the stringy phase the average time evolution of the coupling constant can be described by:

$$a(\eta) \simeq \eta^{-1}, \quad g(\eta) \simeq |\eta|^{-\beta}, \quad \eta_s < \eta < \eta_r, \tag{28}$$

where $\beta = -(\phi_s - \phi_r)/(2\ln z_s)$ where $z_s = \eta_s/\eta_r$. For $\eta > \eta_r$, the background is dominated by radiation (i.e. $a(\eta) \simeq \eta$) and the coupling constant freezes to its constant value (i.e. $\phi = \phi_r =$ const. for $\eta > \eta_r$). Notice that $g(\eta_r) = \exp(\phi_r/2) = g_r \simeq 0.1$–$0.01$. For $k\eta < 1$ we have, from Eqs. (25) that

$$\overline{n}_k(\eta_r) \simeq |\eta_i/\eta_r|^{2\beta} \sim |k/k_r|^{-2\beta}, \quad k_s < k < k_r \tag{29}$$

where $|\eta_i| \sim k^{-1} < |\eta_s|$. Similarly, if $|\eta_i| > |\eta_s|$,

$$\overline{n}_k(\eta_r) \simeq |k/k_s|^{-\sqrt{3}} |g(\eta_s)/g(\eta_r)|^{-2}, \quad k < k_s \quad . \tag{30}$$

Notice that we assumed $\beta > 0$ which means that the coupling constant does not decreases during the stringy phase. Due to magnetic flux conservation [10, 11] the fraction of electromagnetic energy stored in the mode ω does not change and it is defined as

$$\lambda(\omega) = \frac{1}{\rho_\gamma}\frac{d\rho_B}{d\ln\omega} = \frac{g^2}{4\pi}\left(\frac{\omega}{\omega_r}\right)^4 \overline{n}_k(\eta_r)\sin^2 k\eta, \tag{31}$$

$$\rho_\gamma(\eta) = M_P^2 H_r^2 \left(\frac{a_r}{a}\right)^4 \equiv \omega_r^4 \left(\frac{g_r}{4\pi}\right)^2 \tag{32}$$

where $\omega_r \sim a_r/\eta_r = \sqrt{g_r/4\pi}10^{11}$ Hz is the maximal amplified frequency red-shifted today and where we assumed $\overline{n}_k(\eta_r) > 1$. Notice that in the unifying notation of eq. (31) the oscillating part occurs, for each mode, when $k\eta > 1$ but not in the opposite limit where $\phi_k \sim \pi/2$. The critical density constraint, implies, during the stringy phase that $\beta < 2$. If $\beta \lesssim 2$ (for instance $\beta \simeq 1.9$) we can have that $\lambda(\omega_{\rm dec}) = g_r^2(\omega_{\rm dec}/\omega_r)^{4-2\beta} \sim 10^{-8}$ for $\omega_{\rm dec} \sim 10^{-16}$ Hz (for $\omega_{\rm dec} > \omega_s$). Recall that, in order to rotate the polarization plane of the Cosmic Microwave Background Radiation, we need, at decoupling, $B \gtrsim 10^{-3}$ Gauss, or in our language, $\lambda(\omega_{\rm dec}) \gtrsim 10^{-8}$ [16]. Similarly, at the scale of 1 Mpc (i.e. $\omega_G \sim 10^{-14}$ Hz) we can have $\lambda(\omega_G) > 10^{-10}$ [7]. Both at the galactic and decoupling frequencies $\overline{n}_\omega \gg 1$ in the framework of this specific model and the quantum mechanical state is strongly squeezed ($|r_k| \gg 1$).

The quantum degree of second order coherence is a measure of the correlation of the magnetic field intensities at two space-time points. The intensity fluctuations of a given light field are described by the Glauber correlation function [12] which is nothing but the quantum mechanical generalization of the correlation between the classical intensities of two light beams. In quantum optics one deals mainly with intensity correlations of electric fields. This is due to the fact that the photons of the visible part of the electromagnetic spectrum are detected via photo-electric effect, and, therefore, what is indeed detected is an electric current induced by a photon. In our case we are mainly interested in the intensity correlations of the magnetic part of the field and we can write the corresponding correlation $\Gamma(\vec{r})$ of the intensity operators as

$$\frac{\langle : \hat{\beta}^-(\vec{x},\eta)\ \hat{\beta}^-(\vec{x}+\vec{r},\eta)\ \hat{\beta}^+(\vec{x}+\vec{r},\eta)\ \hat{\beta}^+(\vec{x},\eta) : \rangle}{\langle : \hat{\beta}^-(\vec{x},\eta)\hat{\beta}^+(\vec{x},\eta) : \rangle \langle : \hat{\beta}^-(\vec{x}+\vec{r},\eta) : \rangle}. \tag{33}$$

In this (normal ordered) definition the field operators refer to a single polarization of the field, namely

$$\hat{\beta}^{+}(\vec{x},\eta) = \frac{ig}{(2\pi)^{3/2}}\int d^3k\sqrt{\frac{k}{2}}\hat{a}_k(\eta)e^{i\vec{k}\cdot\vec{x}}, \tag{34}$$

with $\hat{\beta}^{-} = (\hat{\beta}^{+})^{\dagger}$. $\Gamma(\vec{r})$ describes correlation between intensities in the case where the photons are detected simultaneously in time but at different spatial locations. The statistical properties of the given quantum mechanical state of the field are encoded in the intercept of the Glauber function namely $\Gamma(0)$.

By using Eqs. (9) into Eq. (33) we obtain

$$\begin{aligned}\Gamma(\vec{r}) &= \frac{\int d^3k\ k|\nu_k(\eta)|^2 \int d^3p\ p|\nu_p(\eta)|^2}{\int d^3k\ k|\nu_k(\eta)|^2\int d^3p\ p|\mu_p(\eta)|^2} \\ &+ \frac{\int d^3k\ k|\nu_k(\eta)|^2 \int d^3p\ p|\nu_p(\eta)|^2 e^{i(\vec{k}-\vec{p})\cdot\vec{x}}}{\int d^3k\ k|\nu_k(\eta)|^2\int d^3p\ p|\mu_p(\eta)|^2} \\ &+ \frac{\int d^3k\ \int d^3p\ p\ k\ \nu_k^*(\eta)\mu_k^*(\eta)\nu_p(\eta)\mu_p(\eta)e^{i(\vec{k}-\vec{p})\cdot\vec{r}}}{\int d^3k\ k|\nu_k(\eta)|^2\int d^3p\ p|\mu_p(\eta)|^2}\end{aligned} \tag{35}$$

which in the limit $|\vec{r}| \to 0$ becomes

$$\Gamma(0) = 2 + \frac{\int k\ d^3k \int p\ d^3p\nu_k^*(\eta)\mu_k^*(\eta)\nu_p(\eta)\mu_p(\eta)}{\int d^3k\ k|\nu_k(\eta)|^2\int d^3p\ p|\nu_p(\eta)|^2}. \tag{36}$$

In order to interpret this formula we can introduce a further simplification, namely we can restrict our attention to a single mode of the field. Then, $\mu_k(\eta) \to \mu_K(\eta)\delta^{(3)}(\vec{k}-\vec{K})$ and $\nu_k(\eta) \to \nu_K(\eta)\delta^{(3)}(\vec{k}-\vec{K})$. Thus $\Gamma(0) = 3 + |\overline{\nu}_K|^{-2}$. As we showed, large variations in $g(\eta)$ give $|\nu_K|^2 \gg 1$. In the case of a coherent state the intercept of the Glauber function is exactly one, namely $\Gamma(0) = 1$ [12]. This property is in direct correspondence with the Poissonian character of the statistics. In the case of a thermal state (i.e. "white light"), $\Gamma(0) \to 2$ [12]. In the case of squeezed relic photons for large number of particles in each Fourier mode $\Gamma(0) \to 3$. Since $\Gamma(0)$ represents the probability of two photons arriving at the same location, this is referred to as photon bunching. Conversely, a field with sub-Poissonnian statistics will have $\Gamma(0) < 1$, an effect known as photon anti-bunching in the context of the Hanbury Brown-Twiss interferometry [17].

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